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COMPUTATION OF BEAM PATTERNS AND DIRECTIVITY INDICES FOR THREE-DIMENSIONAL ARRAYS WITH ARBITRARY ELEMENT SPACINGS

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The computation of beam pattern functions involves the summation of individual element responses that contain spatial and artificially induced phase delays. In this manner, the total array response to acoustic wave excitation possessing directionality characteristics can be determined. A computer program that calculates the beam pattern function for any selected section of the beam pattern is discussed.

Some special features of the program are: both uniform and nonuniform spac-

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20. (Cont'd).

ing of hydrophones can be accommodated; either plane or spherical wave excitation can be assessed; the beam pattern can be determined for any desired array steering direction; any desired element response function can be incorporated into the beam computations; any prescribed amplitude shading technique can be applied to the array; and arbitrary single-element phase delays can be introduced into the beam computations. The latter feature is useful for assessing the effects on array performance due to errors in the computed steering delays, errors in the locations of hydrophones, or errors of random phase components in propagating wave energy intercepted by the array.

The output format is selectable and can be either a polar or a rectangular representation. In addition to the beam patterns, the program also determines values of directivity index and reverberation index.

PREFACE

REVIEWED AND APPROVED: 22 February 1974

R. M. Dunlap
Director, Plans and Analysis

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COMPUTATION OF BEAM PATTERNS AND DIRECTIVITY INDICES FOR THREE-DIMENSIONAL ARRAYS WITH ARBITRARY ELEMENT SPACINGS

INTRODUCTION

A special formulation of the computation of beam pattern functions of arbitrary arrays is presented herein, along with some computational aspects. A package of computer programs has been written in FORTRAN V and is described. Complete input parameter definitions, output information, usage, and program descriptions are given in detail in the text and in the tables. Selected numerical examples exhibit the different options and show the typical results obtained with the beam patterns being described graphically and the directivity index against noise and reverberation being given numerically in decibels. The computations associated with the program are carried out on the Univac 1108 computer in single-precision arithmetic with the EXEC 8 system. A listing of the computer programs is included in the appendix.

The package contains some important features which are worthy of note. First, the design allows treatment of arrays whose elements are distributed in three-dimensional space in a completely arbitrary fashion, with either uniform or nonuniform spacing between elements. Determination of beam pattern functions is computed in a manner that allows the beam to be defined in any plane surface which passes through the array center. By a selective sectioning in angular spherical coordinates, this technique can produce representations of three-dimensional beam shapes.

The program provides for array response to either plane or spherical wave excitation. In addition, in order to treat completely arbitrary arrays, an individual array element response is incorporated which has directional receiving properties. Further, the program is capable of incorporating any amplitude shading technique desired. Arbitrary phase delays in element response may also be added, apart from the usual artificially introduced array steering delays. This additional feature is useful, for example, in determining array performance when unwanted or uncompensated phase delays are present in the system.

Directivity index and reverberation index are computed from beam pattern functions by the application of numerical quadrature techniques. An option is provided for integration in subintervals with different mesh sizes in order to minimize computer running time.

1

MATHEMATICAL FORMULATION

BEAM PATTERN FUNCTION

The beam pattern function $b(\theta, \phi)$ is considered to be dependent on the two spherical coordinate deviation angles: θ, defining the azimuthal deviation in the horizontal plane, and ϕ, the vertical deviation from a horizontal reference. The reference direction in azimuth may generally be assigned to coincide with a geometric axis of the array if, as is usually the case, there is inherent symmetry in the physical array structure. The origin of coordinates is assumed to be the geometric center of the set of elements comprising the array, as shown in figure 1. The array element is designated by its Cartesian position (x_k, y_k, z_k) $k = 1, 2, \ldots, N$.

ARRAY RESPONSE TO PLANE WAVE EXCITATION

A plane wave that is impinging on the array is completely defined, insofar as its effect on the output of the array is concerned, by the direction of propagation (θ, ϕ) of the wave and its wavelength λ. The direction perpendicular to the incoming wavefront is defined by direction cosines, $\cos \alpha$, $\cos \beta$, and $\cos \gamma$, where

$$\cos \alpha = \cos \phi \sin \theta$$

$$\cos \beta = \cos \phi \cos \theta \tag{1}$$

$$\cos \gamma = -\sin \phi.$$
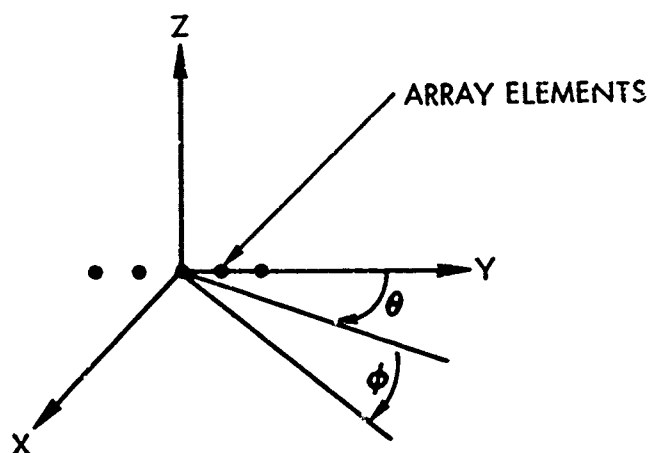
Figure 1.   Coordinate System for Array
Structure and Beam Pattern Function

A reference plane for measuring relative phase at any element of the array is chosen to be that plane passing through the origin which is parallel to all wavefronts, i.e.,

$$x \cos \alpha + y \cos \beta + z \cos \gamma = 0 \ .$$

The distance from any element to the reference plane is then

$$d_k = x_k \cos \alpha + y_k \cos \beta + z_k \cos \gamma \qquad (k = 1, 2, \ldots, N) \tag{2}$$

and the phase of the plane wave excitation at the same element relative to that corresponding to the reference plane is

$$\Phi_k = \frac{2\pi}{\lambda} d_k \ . \tag{3}$$

The "voltage" output of the unsteered array for plane wave excitation is then given by

$$V(\theta, \phi) = \sum_{j=1}^{N} R_j e^{i(2\pi/\lambda)d_j} \ , \tag{4}$$

where R_j is the amplitude response of the j-th element. Phase delays corresponding to preassigned array steering angles can be readily incorporated into the voltage function. If (θ_0, ϕ_0) are the prescribed steering directions, the equivalent "delay length" is given by

$$D_k = x_k \cos \alpha_0 + y_k \cos \beta_0 + z_k \cos \gamma_0 \ , \tag{5}$$

where $\cos \alpha_0$, $\cos \beta_0$, $\cos \gamma_0$ are the direction cosines of the vector corresponding to the steering directions. The voltage output of the steered array is then

$$V(\theta, \phi) = \sum_{j=1}^{N} R_j e^{i(2\pi/\lambda)(d_j - D_j)} \ . \tag{6}$$

3

The beam pattern function in decibels is defined as minus the logarithm of the ratio of beam output power relative to the maximum output power over all directions,

$$b(\theta,\phi) = -10 \log_{10} \frac{\left|V^2(\theta,\phi)\right|}{\left|V^2(\theta,\phi)\right|_{max}} \quad 0 \leq \theta \leq 2 \; , \quad -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2} \; . \qquad (7)$$

In the event that the array is steered, the denominator in the previous expression corresponds to the array response in the direction of the steering vector.

ARRAY RESPONSE TO SPHERICAL WAVE EXCITATION

At times it may be of interest to examine the array beam pattern function when the impinging wave does not have plane fronts. In this event, an alternative form is chosen as a spherical wave. Thus it is possible to examine the array responses under other than farfield source conditions. Let the symbol r_0 specify the distance of a point source from the array center; then if (θ,ϕ) prescribes the angular orientation of the source, its Cartesian position coordinates are

$$x_\xi = r_0 \cos\phi \sin\theta$$

$$y_\xi = r_0 \cos\phi \cos\theta \qquad (8)$$

$$z_\xi = r_0 \sin\phi \; .$$

The spherical wavefront passing through the origin is given by the equation

$$(x - x_\xi)^2 + (y - y_\xi)^2 + (z - z_\xi)^2 = r_0^2 \; . \qquad (9)$$

The distance from an arbitrary element along a radial direction from the source to this reference wavefront is easily found to be

$$d_k = r_0 - \left[(x_k - x_\xi)^2 + (y_k - y_\xi)^2 + (z_k - z_\xi)^2 \right]^{1/2} \; . \qquad (10)$$

The "delay lengths" for steering directions (θ_0, ϕ_0) are found from equation (5), since it may usually be assumed that the array beam would be steered for plane wave excitation.

DIRECTIVITY INDEX

The directivity index (DI) is expressed by the formula

$$DI = -10 \log_{10} \frac{\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \left| V^2(\theta,\phi) \right| \cos\phi \, d\phi \, d\theta}{4\pi \cdot \left| V^2(\theta,\phi) \right|_{max}} . \tag{11}$$

REVERBERATION INDEX

The reverberation index (RI) is expressed by the formula

$$RI = -10 \log_{10} \frac{\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} RV(\theta,\phi) \left| V^2(\theta,\phi) \right| \cos\phi \, d\phi \, d\theta}{4\pi \cdot \left| V^2(\theta,\phi) \right|_{max}} , \tag{12}$$

where $RV(\theta,\phi)$ is a reverberation function which is used to effectively describe the relative reverberation strength of backscattered energy from boundary or volume scatterers over the angular intervals involved.

COMPUTATIONAL ASPECTS

The computer program developed to perform array beam calculations contains three separate sections:

1. Determination of the array beam pattern

2. Evaluation of DI

3. Evaluation of RI for an arbitrary reverberation function.

Given the physical structure of the array, the first of these merely involves performing the summation given in equation (6) for either the plane or the spherical wave. Provision is made to insert the individual element response and a possible coefficient for amplitude shading from two separate subroutines of the CALL statement. Similarly, the physical delays as well as those artificially introduced for array steering are computed in another such subroutine. The computed beam function is presented as either a polar or a rectangular plot, as required by the user. Details of input information and use of the program are given in later sections.

The second function, computation of directivity index, involves integration of a two-dimensional integral of the form

$$I = \int_a^b \int_c^d f(x, y) \; dxdy \; . \tag{13}$$

The numerical evaluation of the above integral is accomplished by means of a quadrature formula,

$$I \approx \sum_{i=1}^M \; \sum_{j=1}^N \; H_i H_j \; f(x_i, y_j) \; . \tag{14}$$

The third program function is similar to the above, since computation of the reverberation index also involves the quadrature formula. In addition, it is necessary to provide a reverberation function which is to be generated in a subroutine provided by the user.

The integration mesh size is a two-dimensional variable in order to economize on computation time. In the absence of round-off errors, a sufficiently small mesh size enables the quadrature formula to adhere to a certain desired computation accuracy.

DEFINITION OF INPUT PARAMETERS

The input parameters appearing on the NAMELIST of the executive program have four different categories:

1. ARRAY CHARACTERISTICS

2. BEAM PATTERN FUNCTIONAL CONTROL PARAMETERS

3. DI AND RI COMPUTATIONAL CONTROL PARAMETERS

4. CONTROL PARAMETER OF SEQUENTIAL RUNS.

They are defined below.

1. ARRAY CHARACTERISTICS

DIMENSION INDEX

L	Number of elements in the x-direction
M	Number of elements in the y-direction
N	Number of elements in the z-direction
LMN	Total number of elements (for irregular element spacing)

FREQUENCY (Hz)

F　　　　　　Frequency

SOUND VELOCITY C

VELCTY　　If unspecified, the "default" value is 4900.0 ft/sec

SPACING BETWEEN ELEMENTS

IXYZ = 0　　Implies elements equally spaced in x, y, and z directions

XSPACE　　Spacing between elements in x-direction (ft)

YSPACE　　Spacing between elements in y-direction (ft)

ZSPACE　　Spacing between elements in z-direction (ft)

**A nonzero indicator IXYZ requires that each element location be given in three-dimensional Cartesian coordinates. These can be given in free format.

SELECTABLE PHASE DELAY OPTION

IDELAY = 0　Implies no preselected phase delay associated with elements.

**If IDELAY is not 0, DELAY subroutine should be supplied with selectable delay information

ARRAY ELEMENT DIRECTIONALITY

IR = 0　　Implies nondirectional elements; otherwise the subroutine RSPNSE has to be supplied, containing the information of directionality in the array R

SHADING TECHNIQUE

ISHAD = 0　Implies no shading of individual element.

**Nonzero ISHAD requires an amplitude shading technique to be given in SHADE subroutine

WAVEFRONT

RZERO $\leq$ 0　Plane wavefront

RZERO > 0　Spherical wavefront

**RZERO is the distance from a source to the origin

7

MAXIMUM ARRAY RESPONSE ON MAJOR AXIS

MRA	Set = 0 initially.

The steering direction must be given to determine the maximum response. Once the maximum response value is determined, the program automatically sets $MRA \neq 0$. For subsequent runs, the latest maximum response value is used, unless a different steering direction is specified with MRA = 0.

THEM	Horizontal steering direction angle
PHIM	Vertical steering direction angle

2. BEAM PATTERN FUNCTION CONTROL PARAMETERS

BEAM TYPE

IBEAM = 0	Horizontal beam
IBEAM = 1	Vertical beam
IBEAM > 1	Calculates major response value; implies no need to calculate the beam pattern; calculations proceed directly to the computation of DI/RI

PLOTS

IPLOT < 0	Plot is suppressed
IPLOT = 0	Rectangular plot
IPLOT > 0	Both rectangular and polar plots

PLOT SCALES

ALONG	Rectangular plot, horizontal length in inches
DLLONG	Rectangular plot, vertical scale in decibels
DIAM	Polar plot, radius in inches
PLDB	Polar plot, scale in decibels

CONTROL OF NUMERICAL OUTPUTS
OF BEAM PATTERN FUNCTION

IPRINT = 0	Printout of numerical values is suppressed

ANGULAR LIMITS AND INCREMENTS (Degrees)

THMIN   Minimum horizontal angle for beam computation

THMAX   Maximum horizontal angle for beam computation

PHIMIN   Minimum vertical angle for beam computation

PHIMAX   Maximum vertical angle for beam computation

DTHETA   Horizontal angular increment between successive computations

DPHI   Vertical angular increment between successive computations

3. DI AND RI: COMPUTATIONAL CONTROL PARAMETERS

CONTROL OF DI COMPUTATION

$INDEX = 0$ Implies that the calculation of DI is to be omitted

CONTROL OF RI COMPUTATION

$IREV \neq 0$ The calculation of RI is required. The reverberation function $RV(\theta, \phi)$ must be given in a subroutine REVB

INTEGRATION LIMITS AND SPACING

HHZ   Horizontal integration step size

Default value = 3°

HVT   Vertical integration step size

Default value = 3°

LOWH   Lower limit of integration in horizontal direction

Default value = 0

LIMH   Number of points considered between horizontal limits

Default value = 120

LOWV   Lower limit of integration in vertical direction

Default value = −90°

LIMV   Number of points considered between vertical limits

Default value = 60

INTEGRATION CONTROL

> MANYH   Number of intervals to be integrated using different step sizes
>
> Default value = 1

4. CONTROL PARAMETER OF SEQUENTIAL RUNS

> ITIME = 0   The very first time entering the program. (The program initializes the quantity to be 0)
>
> ITIME = 1   Implies there is no need to repeat the calculations of the array elements, the element response, delay, shading, and the Major Response.
>
> ITIME > 1   Implies there is no need to calculate the beam pattern function, plots, and related computations.

Other than the control parameter of sequential runs, the foregoing tabulation of input parameters is summarized in tables 1, 2, and 3.

Table 1. Array Characteristics

Description	Symbol	Default Condition
Dimension Number of elements	L, M, N LMN	
Frequency	F	
Velocity	VELCTY	4900.0 ft/sec
Elements spacing	IXYZ XSPACE YSPACE ZSPACE	
Delay	IDELAY DELAY DELA	0 0.0
Elements response	IR RSPNSE R	0 1.0
Shading technique	ISHAD SHADE SHAD	0 1.0
Wavefront	RZERO	0.0
Array response on major axis	MRA	0

Table 2.   Beam Pattern Function
Control Parameters

THMIN $= \theta_{min} \leq \theta \leq \theta_{max} =$ THMAX		
PHIMIN $= \phi_{min} \leq \phi \leq \phi_{max} =$ PHIMAX		
Description	Symbol	Default Condition
	IBEAM	Horizontal
	IPLOT	0
	IPRINT	0
θ_{min}	THMIN	0
θ_{max}	THMAX	360°
ϕ_{min}	PHIMIN	-90°
ϕ_{max}	PHIMAX	90°
$\Delta\theta$	DTEETA	10°
$\Delta\phi$	DPHI	10°

Table 3.   DI and RI Computation
Control Parameters

The following integral is considered.		
$$\int_a^b \int_c^d f(\theta,\phi)\, d\phi\, d\theta$$		
Description	Symbol	Default Condition
a	LOWH	0
b	LIMH	2π
c	LOWV	$-\pi/2$
d	LIMV	$\pi/2$
$\Delta\theta$	HHZ	3°
$\Delta\phi$	HVT	3°
	MANYH	1

PROGRAM DESCRIPTION

The overall structure of the computer program is described by means of table 4, the following brief outline of the executive program, and the various subroutines controlled by the program.

Table 4. Structure of the
Computer Program

```
GBEAM
     |———— SUM
     |———— XYZ
     |———— RSPNSE
     |———— DELAY
     |———— SHADE
     |———— BPLOTR
     |———— BPLOTP
     |          |————RADIAL
     |———— REVP
```
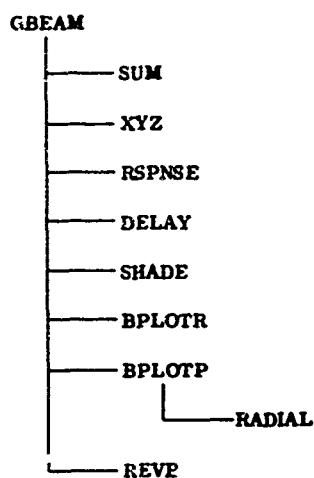
GBEAM — This is the executive program that commands the specific perform-
ance requested by the user. It performs the following functions:

1. Accepts input data

2. Calculates the wavelength

3. Calls RSPNSE, DELAY, SHADE, if they are required

4. Commands the setup of element spacing, if equal spacing is required

5. Lists certain input information

6. Displays some processed information

7. Determines Major Response of array

8. Calls for rectangular and/or polar plots

9. Calculates horizontal and/or vertical beams

10. Calculates DI

11. Calculates RI

12. Prints out numerical results.

The various subroutines are:

SUM   Calculates Major Response and the delays

XYZ   Calculates the element locations, if equally spaced, and accepts irregularly spaced array element locations provided as input to the program

RSPNSE   Obtains element response, if elements do not possess an omnidirectional character

DELAY   Accepts selectable delays

SHADE   Accepts an arbitrary amplitude shading technique

BPLOTR   Generates a rectangular plot

BPLOTP   Generates a polar plot

RADIAL   Sets up radial scale for polar plot

REVB   Describes a reverberation function.

PROGRAM USE

The coding of the program for the Univac 1108 computer is compatible with the EXEC 8 system. The program has been labeled GBEAM and is available on tape (Z158, 1st file). Under ordinary use, no additional tapes are needed. The input parameters previously defined should be specified in NAMELIST format under the name INPUTS, and the input procedure should therefore present no great difficulty. Several points, however, require explanation.

In the event that the array spacing is irregular, the proper indication must be given (IXYZ > 0) in the NAMELIST data. When all the NAMELIST data have been entered, the specific locations of all array elements in Cartesian coordinates with the ordering x, y, z should be entered in free format. When the executive program GBEAM calls for a specific element position information, the subroutine XYZ reads the required information and passes this on as required. (When array elements are regularly spaced, the subroutine XYZ calculates the coordinates of the element specified from the given array spacing information.)

The program contains a number of other optional features which require special treatment by the user. Individual elements may be given an amplitude response other than omnidirectional by use of the subroutine RSPNSE, in which the user must define (for example, by means of an equation or tabular data) the desired response function as it depends upon the deviation angles θ and ϕ.

Similarly, an amplitude coefficient compatible with some desired shading technique to the beam may be introduced as a modification to the individual element response by the subroutine SHADE. An example which serves to illustrate the manner in which SHADE subroutine may be used is given in the appendix.

The subroutine DELAY calculates the artificial delay lengths associated with a given tilt and steering angle. Also, it is possible to introduce an arbitrary phase delay into the response of each element by providing, as input, the required numerical values in free format.

The basic options of the program are shown in table 5, which defines the various input quantities according to the option desired. If a reverberation index is desired, the subroutine REVB must be supplied with the reverberation function $RV(\theta, \phi)$ in a manner analogous to that used in RSPNSE, which was previously discussed.

Table 5. Control Parameters

	General Options				
BEAM	•	•	•	•	
DI		•		•	•
RI			•	•	•
THEM	x	x	x	x	x
PHIM	x	x	x	x	x
IBEAM	x	x	x	x	x
ITIME		x	x	x	x
IPLOT	x	x	x	x	
INDEX		x		x	x
IREV			x	x	x
• Items required					
x Items to be specified					

OUTPUT INFORMATION

The first page of the output is given as an aid to the user in checking the input data and consists of the following information:

Title: GENERALIZED BEAM PATTERN PROGRAM

Wavefront type

Array size

Frequency (Hz)/wavelength

Spacing in all directions (ft)

Reference point (geometric center of the array)

Selectable phase delays introduced

Determined array response on major axis.

Subsequent pages contain horizontal or vertical beam pattern numerical outputs if the printout option is selected.

The next output information gives the numerical values of DI/RI if requested.

The last item of output information indicates how many plots will be generated if, indeed, plots have been required.

IMPORTANT FEATURES

Simple mathematical formulas.

Program is general, complete, optional, easy to use, and has no restrictions.

Important parameters are well defined internally. No danger is involved if the user failed to specify some of them. In addition, the user has the option of altering these parameters to satisfy his requirements.

Capability of handling both spherical and plane wavefronts.

Rectangular and polar plots are built in. User can select the plot size.

Automatic setup of equal array element spacing.

Accepts irregularly spaced array elements by their locations.

The use of steering direction information to determine the MRA results in considerable time saving.

Option of accepting the element directionality.

The SHADE subroutine accepts any shading technique.

The program arranges the three-dimensional elements in one-dimensional computations.

Integration limits are adjustable so that the computations can be accelerated. Program can decompose a two-dimensional integral into a set of subintegrals along each subintegral with different step sizes; the step size can be any fraction of one degree.

The listing of the basic input information on the first output sheet enables the user to examine whether or not his input information is supplied correctly.

All subroutines need not be supplied by the user. If they are not called for, the program takes care of them automatically.

Manipulation of the large number of array elements is flexible to the upper limit of computer capacity.

The option of computing and plotting any one section of the beam pattern.

The option of bypassing the beam plots to calculate the DI and/or RI.

If any additional computations are required, the necessary modifications can be incorporated easily.

NUMERICAL EXAMPLES

In this section, selected examples are presented to demonstrate the options of the program and to illustrate typical computation results. To demonstrate subroutine usage, a $\cos^2$ shading technique is applied in one problem. The examples are divided into three categories. The three arrays considered have the configurations described in figure 2.

A LINE ARRAY OF 9 ELEMENTS EQUALLY SPACED WITH HALF-WAVELENGTH SEPARATION

A TWO-DIMENSIONAL EQUALLY SPACED ARRAY, A HALF-WAVELENGTH APART

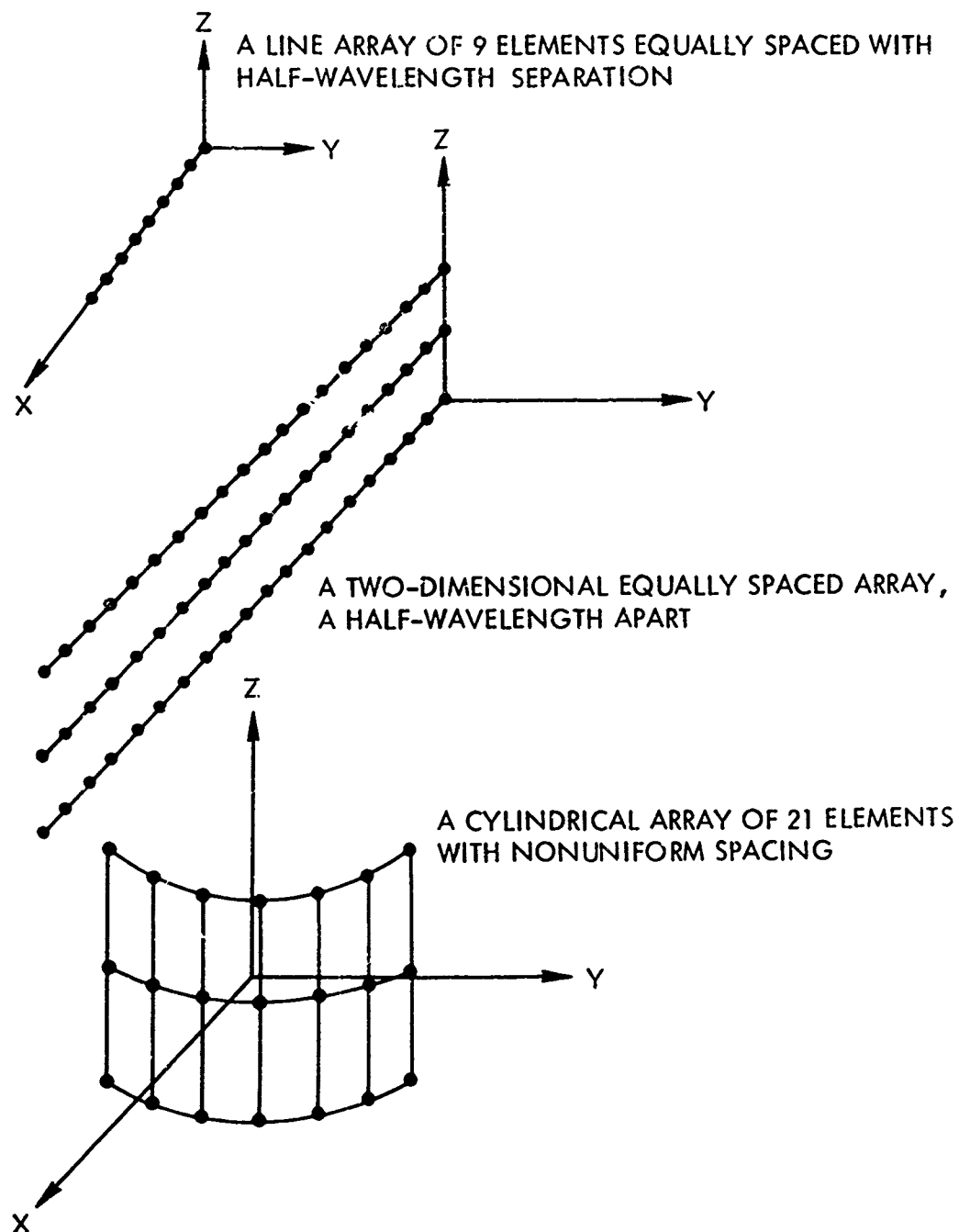A CYLINDRICAL ARRAY OF 21 ELEMENTS WITH NONUNIFORM SPACING

Figure 2.   Array Configurations

Category 1 — The first division has four examples of one-dimensional, equally spaced elements (table 6). Initially, a plane wavefront is considered. Following this, three spherical wavefronts are presented: one has a short distance from the source to the array center; one has a moderate source distance; and one has a great source distance. The effects of the three different source-array separation distances are shown graphically in figures 3 through 6. Each figure consists of rectangular and polar plots of the horizontal and vertical beams for each example. The plots in the figures are identified numerically with their corresponding example numbers in the table.

Category 2 — A two-dimensional, equally spaced rectangular array with a plane wavefront is considered next (table 7). Initially, the beam is computed without shading, then with a $\cos^2$ shading applied in two dimensions. Each example in the table has two different plots. Figures 7 and 8 each contain four plots of either the horizontal or vertical beam patterns for the rectangular and polar plots of each example.

Table 6. Category 1 — One-Dimensional Array (9 x 1 x 1)

Example Number		1.1	1.2	1.3	1.4
Array Excitation	Type	Plane Wave	Spherical Wave	Spherical Wave	Spherical Wave
	Source Distance (ft)	Far Field	5.0	20.0	1000.0
Element Spacing (λ)		0.5	0.5	0.5	0.5
Horizontal Beam	R-plot*	plot 1.1.1	plot 1.2.1	plot 1.3.1	plot 1.4.1
	P-plot†	plot 1.1.3	plot 1.2.3	plot 1.3.3	plot 1.4.3
Vertical Beam	R-plot	plot 1.1.2	plot 1.2.2	plot 1.3.2	plot 1.4.2
	P-plot	plot 1.1.4	plot 1.2.4	plot 1.3.4	plot 1.4.4
Directivity Index		9.54	9.65	9.55	9.54
Reverberation Index		12.55	12.66	12.56	12.55
Computation Time (sec)		27	30	31	32

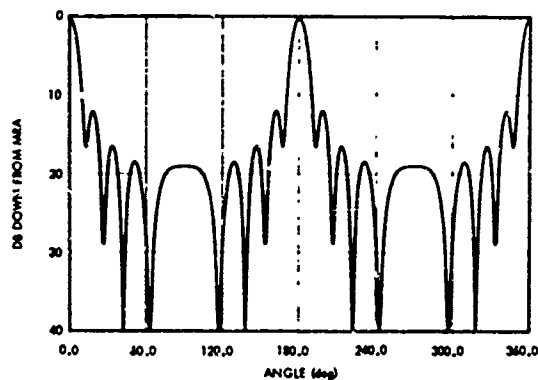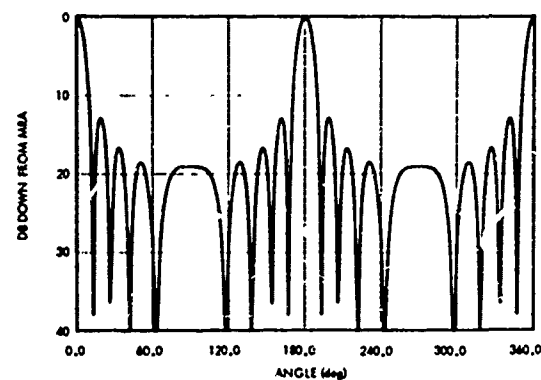*R-plot = rectangular plot
†P-plot = polar plot
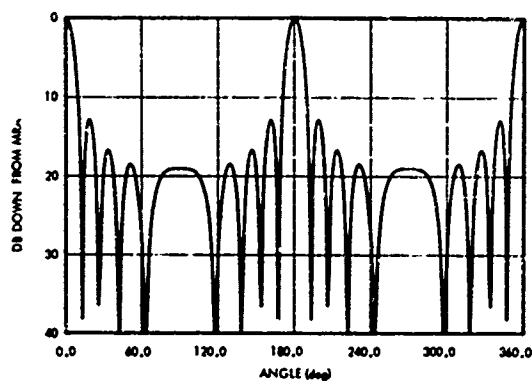
PLOT 1.1.1 LINE ARRAY, PLANE WAVEFRONT
HORIZONTAL BEAM PATTERN
$\phi = 0.0$ deg
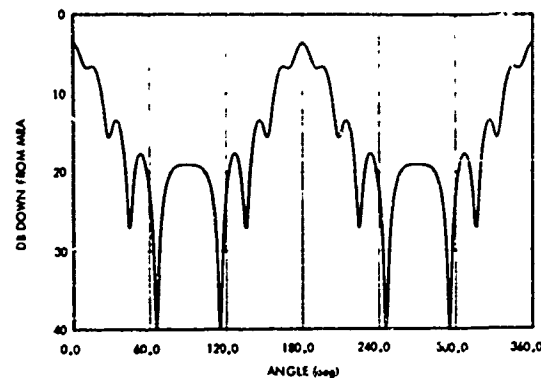
PLOT 1.2.1 LINE ARRAY, SPHERICAL WAVEFRONT (SHORT DISTANCE)
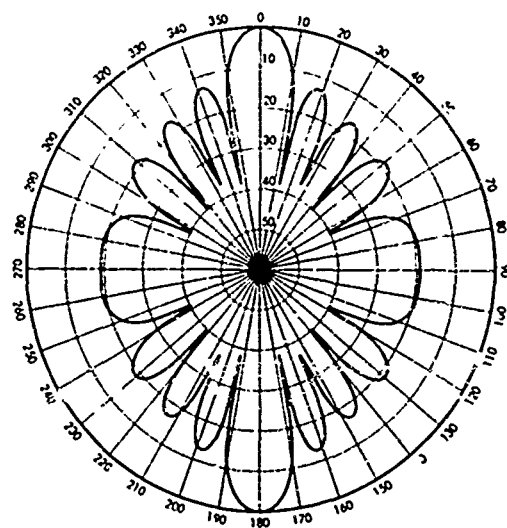HORIZONTAL BEAM PATTERN
$\phi = 0.0$ deg

PLOT 1.3.1 LINE ARRAY, SPHERICAL WAVEFRONT (MODERATE DISTANCE)
HORIZONTAL BEAM PATTERN
$\phi = 0.0$ deg

PLOT 1.4.1 LINE ARRAY, SPHERICAL WAVEFRONT (GREAT DISTANCE)
HORIZONTAL BEAM PATTERN
$\phi = 0.0$ deg

Figure 3.   Horizontal Beam Patterns of Line Array — Rectangular Plots

PLOT 1.1.3 LINE ARRAY, PLANE WAVEFRONT
HORIZONTAL BEAM PATTERN
$\phi = 0.0$ deg

PLOT 1.2.3 LINE ARRAY, SPHERICAL WAVEFRONT (SHORT DISTANCE)
HORIZONTAL BEAM PATTERN
$\phi = 0.0$ deg

PLOT 1.3.3 LINE ARRAY, SPHERICAL WAVEFRONT (MODERATE DISTANCE)
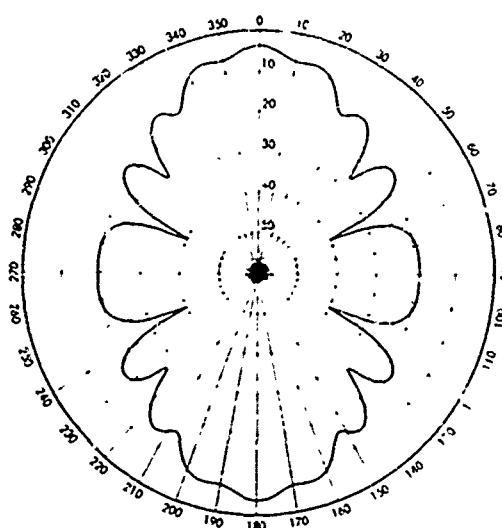HORIZONTAL BEAM PATTERN
$\phi = 0.0$ deg

PLOT 1.4.3 LINE ARRAY, SPHERICAL WAVEFRONT (GREAT DISTANCE)
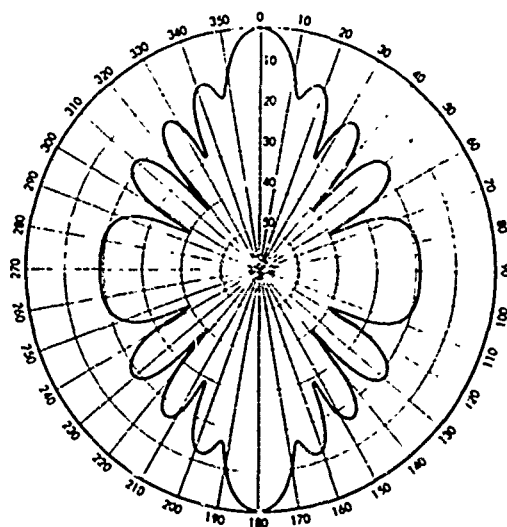HORIZONTAL BEAM PATTERN
$\phi = 0.0$ deg

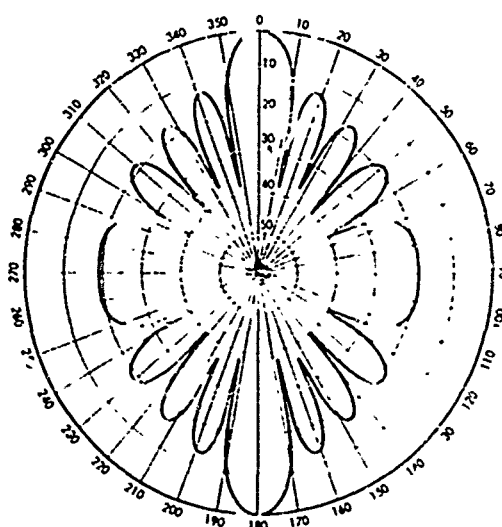Figure 4.   Horizontal Beam Patterns of Line Array — Polar Plots
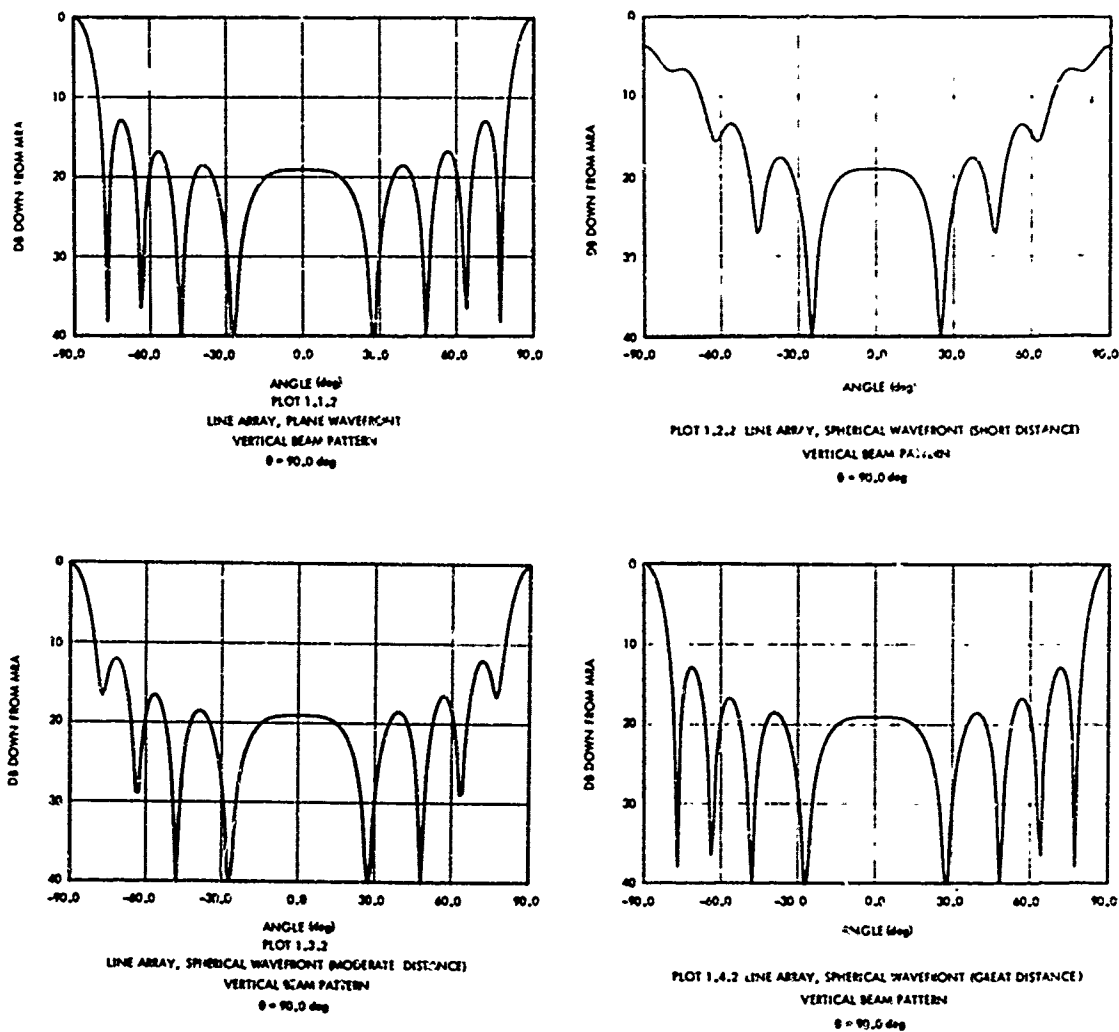
Figure 5. Vertical Beam Patterns of Line Array — Rectangular Plots
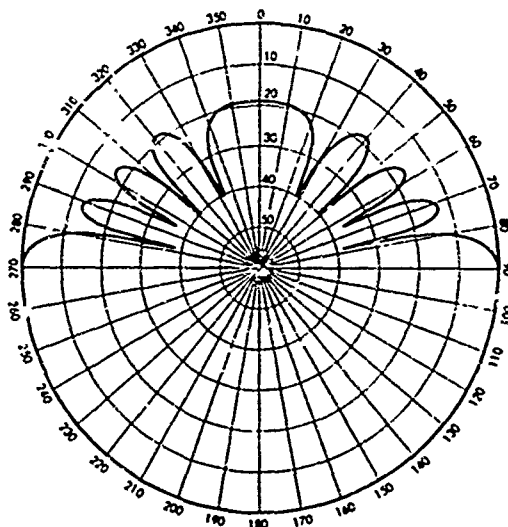
FIGURE 1.1.4 LINE ARRAY, PLANE WAVEFRONT
VERTICAL BEAM PATTERN
θ - 90.0 deg

FIGURE 1.2.4 LINE ARRAY, SPHERICAL WAVEFRONT (SHORT DISTANCE)
VERTICAL BEAM PATTERN
θ - 90.0 deg
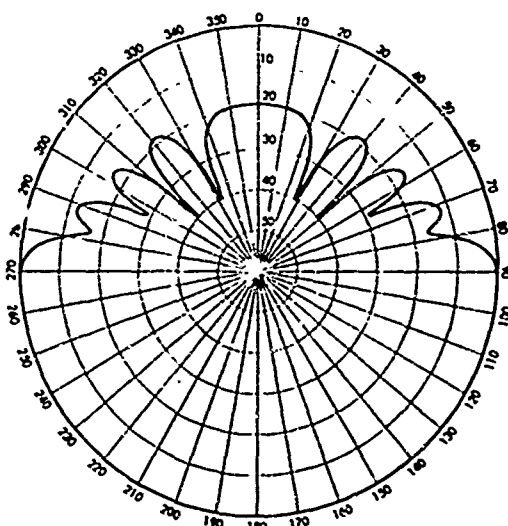
FIGURE 1.3.4 LINE ARRAY, SPHERICAL WAVEFRONT (MODERATE DISTANCE)
VERTICAL BEAM PATTERN
θ - 90.0 deg
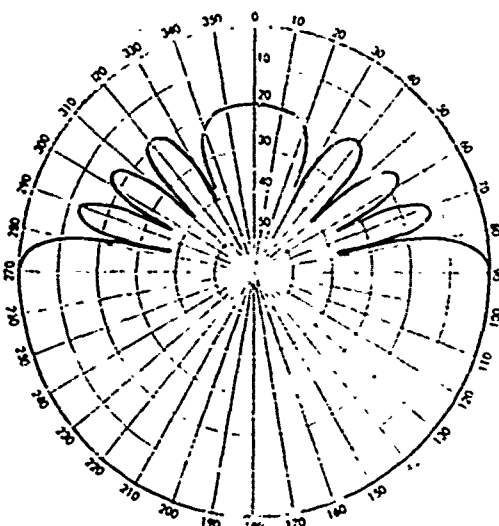
FIGURE 1.4.4 LINE ARRAY, SPHERICAL WAVEFRONT (GREAT DISTANCE)
VERTICAL BEAM PATTERN

Figure 6. Vertical Beam Patterns of Line Array — Polar Plots

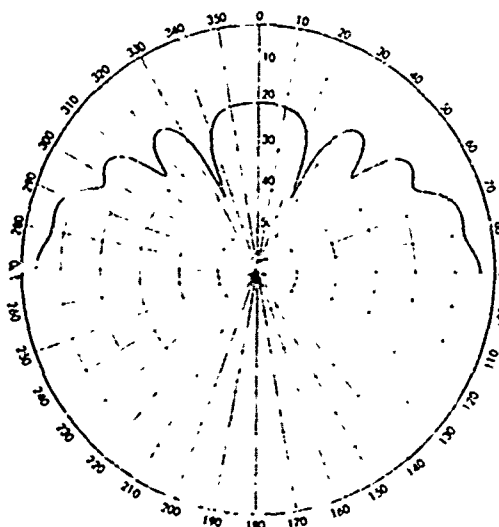Table 7. Category 2 — Two-Dimensional Array (20 x 1 x 3)

Example Number		2.1	2.2
Shading Technique		None	$\cos^2$
Array Excitation	Type	Plane Wave	Plane Wave
	Source Distance (ft)	Farfield	Farfield
Element Spacing (λ)		0.5	0.5
Horizontal Beam	R-plot*	plot 2.1.1	plot 2.2.1
	P-plot†	plot 2.1.3	plot 2.2.3
Vertical Beam	R-plot	plot 2.1.2	plot 2.2.2
	P-plot	plot 2.1.4	plot 2.2.4
Directivity Index	Fixed Step Size	19.03	
	Variable Step Size	19.08	19.00
Reverberation Index	Fixed Step Size	22.04	
	Variable Step Size	22.09	22.01
Computation Time (sec)	Fixed Step Size	101	
	Variable Step Size	58	99
*Rectangular plot †Polar plot			

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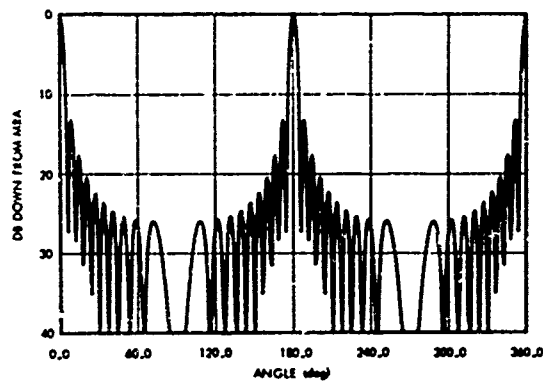
PLOT 2.1.1 RECTANGULAR ARRAY, PLANE WAVEFRONT, NO SHADING

HORIZONTAL BEAM PATTERN
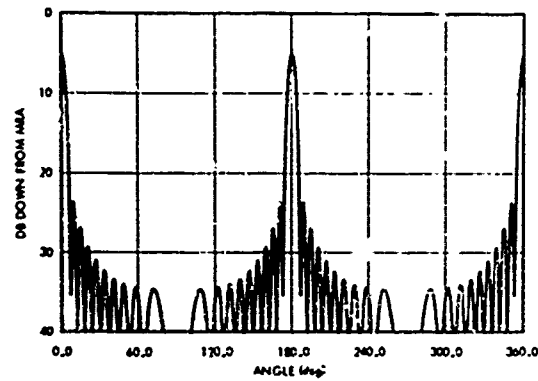
φ = 0.0 deg

PLOT 2.2.1 RECTANGULAR ARRAY, PLANE WAVEFRONT, COS² SHADING

HORIZONTAL BEAM PATTERN
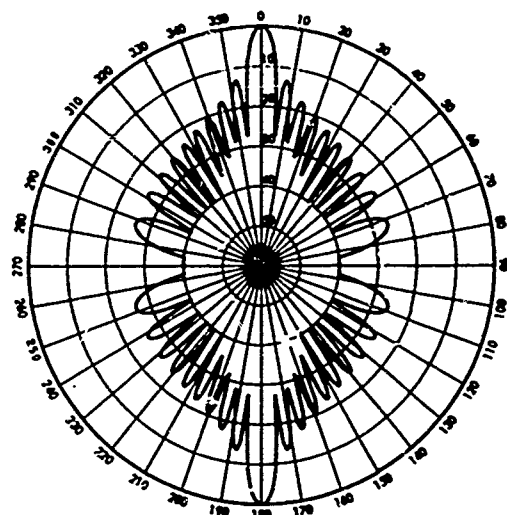
φ = 0.0 deg

PLOT 2.1.3 RECTANGULAR ARRAY, PLANE WAVEFRONT, NO SHADING

HORIZONTAL BEAM PATTERN
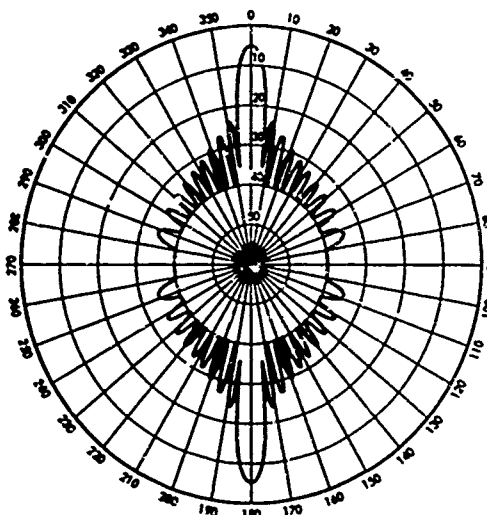
φ = 0.0 deg

PLOT 2.2.3 RECTANGULAR ARRAY, PLANE WAVEFRONT, COS² SHADING

HORIZONTAL BEAM PATTERN

φ = 0.0 deg

Figure 7.   Horizontal Beam Patterns of Rectangular Array
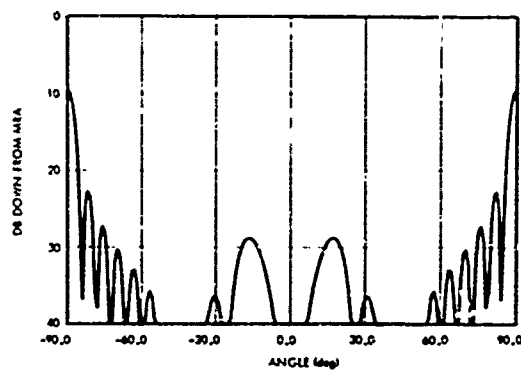
PLOT 2.1.2 RECTANGULAR ARRAY, PLANE WAVEFRONT, NO SHADING
VERTICAL BEAM PATTERN
θ = 90.0 deg
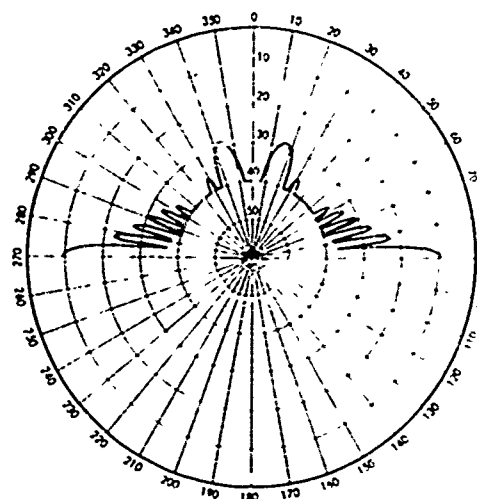
PLOT 2.1.4 RECTANGULAR ARRAY PLANE WAVEFRONT NO SHADING
VERTICAL BEAM PATTERN
θ 90.0 deg
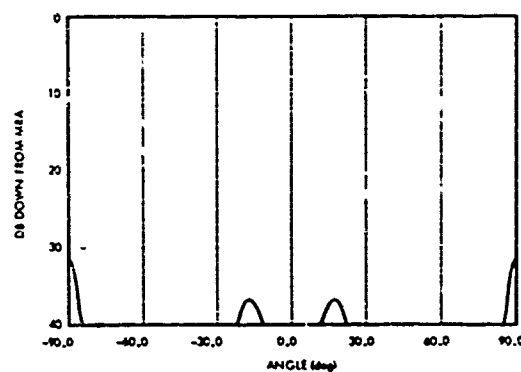
PLOT 2.2.2 RECTANGULAR ARRAY, PLANE WAVEFRONT, COS² SHADING
VERTICAL BEAM PATTERN
θ = 90.0 deg
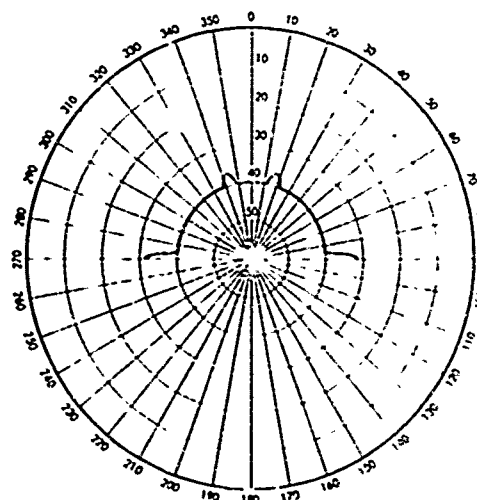
PLOT 2.2.4 RECTANGULAR ARRAY, PLANE WAVEFRONT, COS² SHADING
VERTICAL BEAM PATTERN
θ 90.0 deg

Figure 8. Vertical Beam Patterns of Rectangular Array

Category 3 — Finally, a three-dimensional cylindrical array of 21 elements with different wavefronts is considered (table 8). The cylindrical height = 1λ, the vertical spacing 0.5λ, and the circumferential spacing is $(\pi/6)\lambda$, or roughly 0.5λ, and the radius is 1λ. This category demonstrates the treatment for irregularly spaced elements. In table 8, each example has two different plots. The four plots of figures 9 and 10 show either the horizontal or vertical beam patterns for the rectangular and polar plots of each example.

In the examples, whenever an RI calculation is involved, the function $RV(\theta,\phi)$ is defined for illustrative purposes to be a constant 0.5 for all θ and ϕ. Therefore, no physical significance is intended. Steering directions $\theta = \phi = 0$ are used for examples of categories 1 and 2; $\theta = 45°$, $\phi = 0$ are used for examples of category 3. The distance between elements is chosen to be half-wavelength, if equal spacing is required. Computations of DI and RI are given in tabular form, together with the computation time.

Table 8. Category 3 — Three-Dimensional Array (21 Elements)

Example Number		3.1	3.2
Array Excitation	Type	Plane Wave	Spherical Wave
	Source Distance (ft)	Farfield	5.0
Element Spacing		Irregular	Irregular
Horizontal Beam	R-plot*	lot 3.1.1	plot 3.2.1
	P-plot†	plot 3.1.3	plot 3.2.3
Vertical Beam	R-plot	plot 3.1.2	plot 3.2.2
	P-plot	plot 3.1.4	plot 3.2.4
Directivity Index		13.02	12.80
Reverberation Index		16.03	15.81
Computation Time (sec)		57	54
*Rectangular plot †Polar plot			
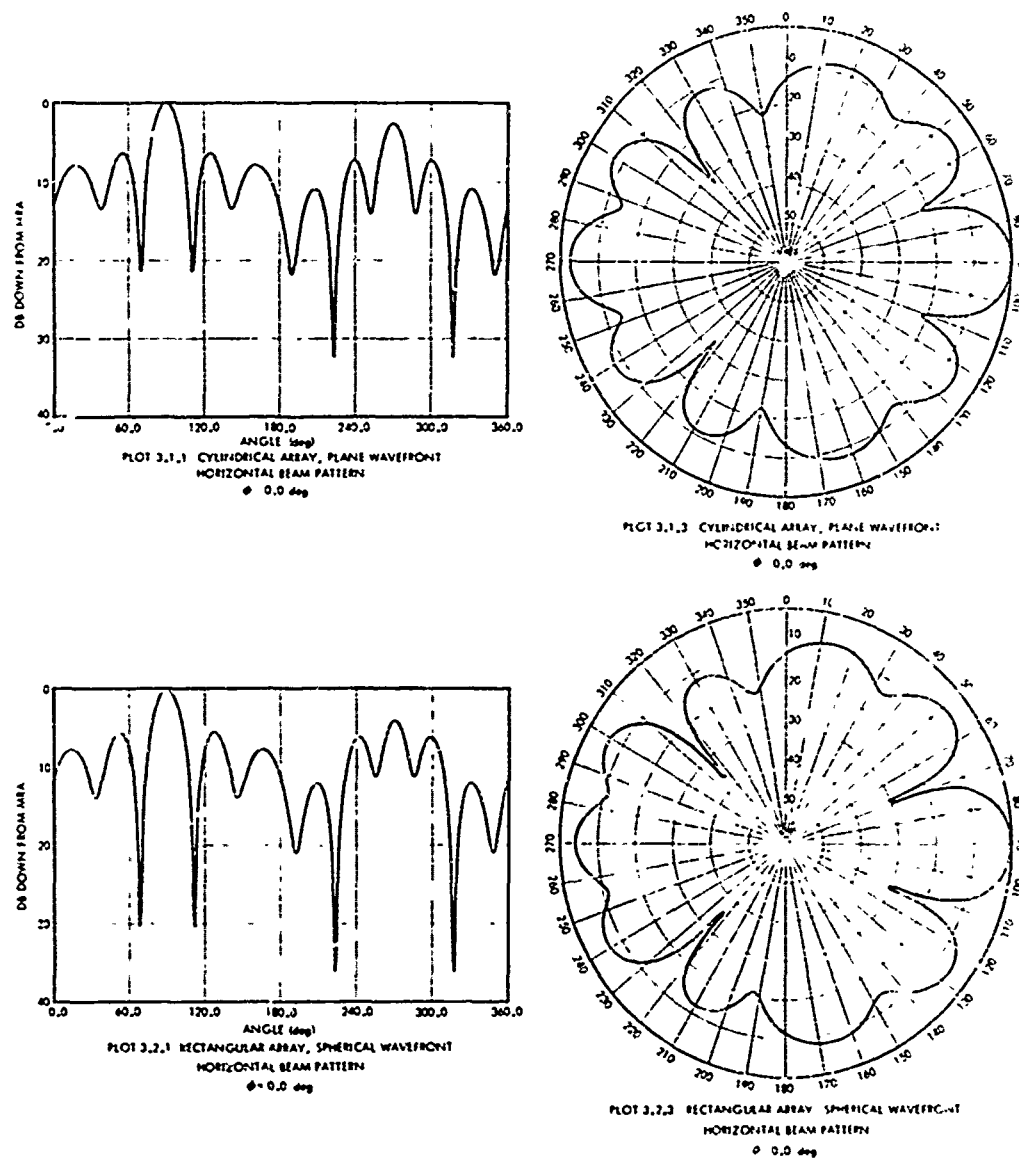
Figure 9. Horizontal Beam Patterns of Cylindrical Array

PLOT 3.1.2 CYLINDRICAL ARRAY, PLANE WAVEFRONT
VERTICAL BEAM PATTERN
θ = 45.0 deg

PLOT 3.1.4 CYLINDRICAL ARRAY, PLANE WAVEFRONT
VERTICAL BEAM PATTERN
θ = 45.0 deg

PLOT 3.2.2 CYLINDRICAL ARRAY, SPHERICAL WAVEFRONT
VERTICAL BEAM PATTERN
θ = 45.0 deg

PLOT 3.2.4 CYLINDRICAL ARRAY, SPHERICAL WAVEFRONT
VERTICAL BEAM PATTERN
θ = 45.0 deg
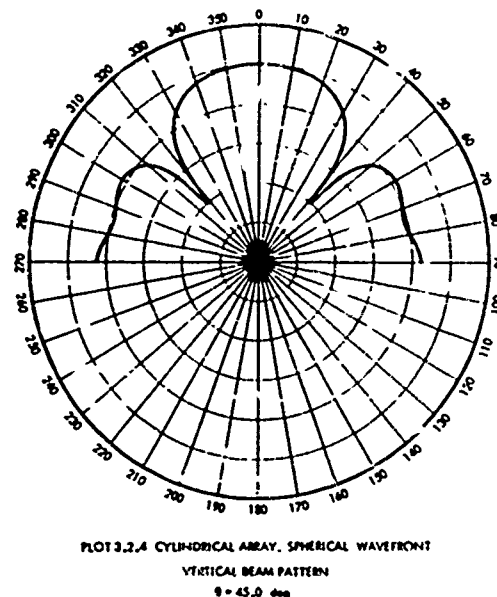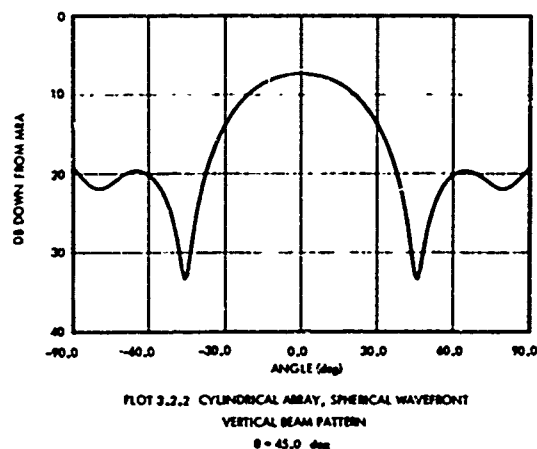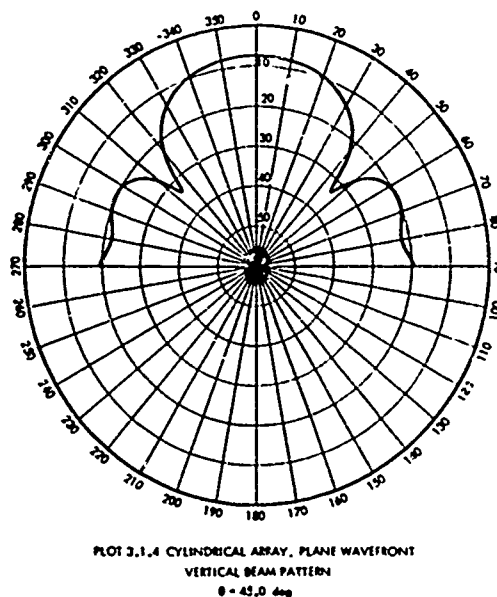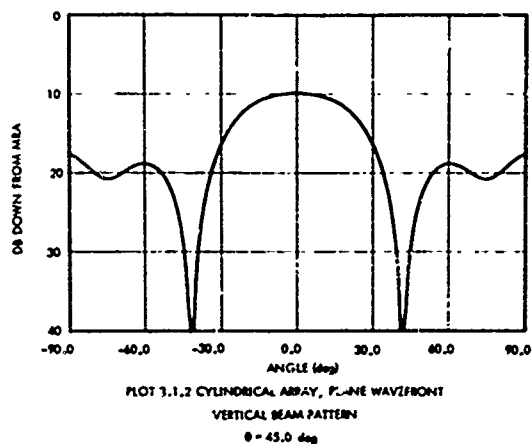
Figure 10.   Vertical Beam Patterns of Cylindrical Array

Example 2.1 of table 7 has two values of DI and RI, which are obtained by using two different mesh sizes for the numerical integration. The value above the line is calculated by integrating horizontally over the interval $(0°, 90°)$, using $1°$ spacing; the value below the line is calculated by the summing of two integrals. One is integrated over the interval $(0°, 15°)$ with a $0.5°$ mesh size; the other is integrated over the interval $(15°, 90°)$ with a $1.5°$ spacing.

REMARKS

The present limit on the total number of elements is 2400. The number of the integration points must be an odd number, i.e., the LIMH, LiMV must be an even number ≥ 2.

REFERENCES

1. P. Davis and P. Rabinowitz, Numerical Integration, Blaisdell Publishing Co., New York, 1967.

2. H. Less, Vector and Tensor Analysis, McGraw-Hill Book Co., Inc., New York, 1950.

3. R. J. Urick, Principles of Underwater Sound for Engineers, McGraw-Hill Book Co., Inc., New York, 1967.

Appendix

COMPUTER PROGRAMS

This section lists all FORTRAN V computer programs required for the computations.

Subroutines RSPNSE and DELAY are made dummies to satisfy the compiler for testing purposes.

Subroutine SHADE describes a pedestal shading technique that is demonstrated in one of the test examples.

GBEAM

```
C **** A GENERALIZED BEAM PATTERN COMPUTATION PROGRAM
C      WITH SHADING, DIRECTIVITY INDEX AND REVERBERATION LEVEL
C **** D.LEE FEB. 1973 - SYSTEMS ANALYSIS DEPARTMENT
C **** IRREGULAR LOCATIONS OF ELEMENTS IN 3-DIMENSION
C **** IR=0 OMNI-DIRECTIONAL
C **** IXYZ=0 ELEMENTS EQUALLY SPACED
C **** FOR SPHERICAL WAVE FRONT, ENTER NONZERO RZERO
C **** IF NOT REQUESTED, HORIZONTAL BEAM IS GIVEN
C      IBEAM=1 =VERTICAL BEAM
      DIMENSION R(2400),XH(2400),YH(2400),ZH(2400)
      DIMENSION DELA(2400),SHAD(2400),BEAM(720)
      DIMENSION PANGLE(720),TANGLE(720)
      DIMENSION HDR(12),DATA(200)
      CALL MODESG(DATA,0)
    1 FORMAT(10X,3(E15.8,15X))
    2 FORMAT(51X,'GENERALIZED BEAM PATTERN PROGRAM'//)
    3 FORMAT(/5X,'ARRAY SIZE',14X,'(',I3,1X,'X',1X,I3,1X,'X',1X,I3,')')
    4 FORMAT(/5X,'FREQUENCY',11X,'=',E15.8,2X,'(HZ)')
    5 FORMAT(/5X,'SPACING Z-DIRECTION =',E15.8,2X,'(FEET)')
    6 FORMAT(/5X,'SPACING X-DIRECTION =',E15.8,2X,'(FEET)')
    7 FORMAT(/5X,'**** HORIZONTAL BEAM ****'//)
    8 FORMAT(/5X,'**** VERTICAL BEAM ****'//)
    9 FORMAT(/5X,'SELECTABLE PHASE-DELAY IS ABSENT')
   10 FORMAT(1H1)
   11 FORMAT(/5X,'SPACING Y-DIRECTION =',E15.8,2X,'(FEET)')
   12 FORMAT(/5X,'SPACING IN X-DIRECTION IRREGULAR')
   13 FORMAT(/5X,'SPACING IN Y-DIRECTION IRREGULAR')
   14 FORMAT(/5X,'SPACING IN Z-DIRECTION IRREGULAR')
   15 FORMAT(40X,2(E15.8,15X))
   17 FORMAT(/5X,'MRA GIVEN AT (THETA=',F6.1,' DEGREES, PHI=',F6.1,' DEG
     *REES)')
   18 FORMAT(12A6)
   19 FORMAT(/5X,'PLANE WAVE FRONT')
   20 FORMAT(/5X,'DIRECTIVITY INDEX   =',F8.2,2X,'(DB)')
   21 FORMAT(//5X,'REVERBERATION LEVEL =',F8.2,2X,'(DB)')
  501 FORMAT(/5X,'SPHERICAL WAVE FRONT -- RO=',E15.8,2X,'(FEET)')
  502 FORMAT(/5X,'LOCATIONS OF',I4,1X,'ARRAY ELEMENTS ARE GIVEN')
  503 FORMAT(/5X,'REFERENCE POINT (',F6.2,1X,',',F6.2,1X,',',F6.2,')')
  504 FORMAT(/5X,'WAVE LENGTH =',F6.2,3X,'(FT)')
      NAMELIST INPUTS/IR,IXYZ,L,M,N,IDELAY,F,RZERO,
     *                XSPACE,YSPACE,ZSPACE,DTHETA,DPHI,IBEAM,THEM,
     *                PHIM,VELCTY,PHIMIN,PHIMAX,THMIN,THMAX,
     *                IPLOT,INDEX,IREV,ITIME,ALONG,DLLONG,ISHAD,
     *                HHZ,HVT,IPRINT,LOWH,LIMH,LOWV,LIMV,MANYH/
      DATA IR,IXYZ,IPLOT,IBEAM,INDEX,IREV,ITIME,MRA,ISHAD/9*0/
      DATA MANYH,IPRINT,LOWH,LIMH,LOWV,LIMV/1,2*0,120,-90,60/
      DATA HHZ,HVT/2*3.0/
      DATA PHIMIN,PHIMAX,THMIN,THMAX,VELCTY/-90.0,90.0,0.0,360.0,4900./
      DATA DTHETA,DPHI,PI,DEG/2*10.0,3.1415926535,.01745329/
      DATA ALONG,DLLONG,PLDB,DIAM/2*6.0,60.0,4.0/
      DATA R,SHAD,DELA/4800*1.0,2400*0.0/
  100 READ (3,INPUTS,ERR=9999,END=9999)
```

```
      READ 18, HDR
      IF(ITIME .NE. 0) GO TO 88
      WLAMDA=VELCTY/F
      CALL XYZ(IXYZ,XH,YH,ZH,L,M,N,XSPACE,YSPACE,ZSPACE,LMN,XO,YO,ZO)
      IF(IR .NE. 0) CALL RSPNSE(LMN,R)
      IF(IDELAY .NE. 0) CALL DELAY(LMN,DELA)
      IF(ISHAD .NE. 0) CALL SHADE(LMN,XH,YH,ZH,XO,YO,ZO)
      RMAX=0.0
      DO 89 I=1,LMN
      IF(RMAX .LT. R(I)) RMAX=R(I)
   89 CONTINUE
   88 CONTINUE
      IF(ITIME .GT. 1) GO TO 300
      IF(ITIME .EQ. 1) GO TO 1111
      WRITE (4,2)
      IF(RZERO .LE. 0.0) WRITE (4,19)
      IF(RZERO .GT. 0.0) WRITE (4,501) RZERO
      IF(L .NE. 0) WRITE (4,3) L,M,N
      IF(L .EQ. 0) WRITE (4,502) LMN
      IF(F .LE. 0.0) WRITE (4,504) WLAMDA
      IF(F .GT. 0.0) WRITE (4,4) F
      IF(IXYZ.EQ.0 .AND. XSPACE.GT.0.0) WRITE (4,6) XSPACE
      IF(IXYZ.EQ.0 .AND. YSPACE.GT.0.0) WRITE (4,11) YSPACE
      IF(IXYZ.EQ.0 .AND. ZSPACE.GT.0.0) WRITE (4,5) ZSPACE
      WRITE (4,503) XO,YO,ZO
      IF(IDELAY .EQ. 0) WRITE (4,9)
      IF(MRA .NE. 0) GO TO 52
      CALL SUM(THEM,PHIM,RZERO,XH,YH,ZH,WLAMDA,LMN,DELA,R,RMAX,ARM,MRA,S
     *HAD)
   52 WRITE (4,17) THEM,PHIM
      WRITE (4,10)
 1111 CONTINUE
      IF(IBEAM.EQ.0 .AND. IPRINT.EQ.0) WRITE (4,7)
      IF(IBEAM.EQ.1 .AND. IPRINT.EQ.0) WRITE (4,8)
      IF(IBEAM .EQ. 0) GO TO 200
      IF(IBEAM .GT. 1) GO TO 300
C **** VERTICAL BEAM
      THETA=THMIN
      JNDEX=0
   30 JNDEX=JNDEX+1
      PHI=PHIMIN+FLOAT(JNDEX-1)*DPHI
      PANGLE(JNDEX)=PHI
      IF(PHI-PHIMAX) 32,32,38
C **** FOR A FIXED THETA, PLOT PHI VS BEAM
   38 JNDEX=JNDEX-1
      IF(IPLOT .LT. 0) GO TO 300
      CALL BPLOTR(IBEAM,PANGLE,BEAM,JNDEX,THETA,PHIMIN,PHIMAX,ALONG,DLLO
     *NG,HDR,DATA)
      IF(IPLOT .GT. 0) CALL BPLOTP(IBEAM,PANGLE,BEAM,JNDEX,THETA,PLDB,DI
     *AM,HDR,DATA)
      GO TO 300
   32 CALL SUM(THETA,PHI,RZERO,XH,YH,ZH,WLAMDA,LMN,DELA,R,RMAX,XMAX,MRA,
     *SHAD)
      BEAM(JNDEX)=-10.*ALOG10(XMAX/ARM)
      IF(IPRINT .NE. 0) GO TO 30
      IF(JNDEX .EQ. 1) GO TO 70
      WRITE (4,15) PHI,BEAM(JNDEX)
      GO TO 30
```

```
   70 WRITE (4,1) THETA,PHI,BEAM(JNDEX)
      GO TO 30
C **** HORIZONTAL BEAM
  200 CONTINUE
      PHI=PHIMIN
      JNDEX=0
  210 JNDEX=JNDEX+1
      THETA=THMIN+FLOAT(JNDEX-1)*DTHETA
      TANGLE(JNDEX)=THETA
      IF(THETA-THMAX) 213,213,214
C **** FOR A FIXED PHI, PLOT THETA VS BEAM
  214 JNDEX=JNDEX-1
      IF(IPLOT .LT. 0) GO TO 300
      CALL BPLOTR(IBEAM,TANGLE,BEAM,JNDEX,PHI,THMIN,THMAX,ALONG,DLLONG,H
     *DR,DATA)
      IF(IPLOT .GT. 0) CALL BPLOTP(IBEAM,TANGLE,BEAM,JNDEX,PHI,PLUB,DIAM
     *,HDR,DATA)
      GO TO 300
  213 CONTINUE
      CALL SUM(THETA,PHI,RZERO,XH,YH,ZH,WLAMDA,LMN,DELA,R,RMAX,XMAX,MRA,
     *SHAD)
      BEAM(JNDEX)=-10.*ALOG10(XMAX/ARM)
      IF(IPRINT .NE. 0) GO TO 210
      IF(JNDEX .EQ. 1) GO TO 230
      WRITE (4,15) THETA,BEAM(JNDEX)
      GO TO 210
  230 WRITE (4,1) PHI,THETA,BEAM(JNDEX)
      GO TO 210
  300 IF(IPRINT .EQ. 0) WRITE (4,10)
C **** TO CALCULATE DIRECTIVITY INDEX & REVERBERATION LEVEL
      IF(INDEX.EQ.0 .AND. IREV.EQ.0) GO TO 100
      TDI=0.0
      TRL=0.0
 1210 IF(MANYH .LE. 0) GO TO 1200
      READ (3,INPUTS,ERR=9999,END=9999)
      FIHOZ=HHZ*DEG
      FIVT=HVT*DEG
      HLOW=FLOAT(LOWH)
      VLOW=FLOAT(LOWV)
      DINDEX=0.0
      RLEVEL=0.0
      IIND=+1
      DO 1000 JO=0,LIMH
      DI=0.0
      RL=0.0
      TH=FLOAT(JO)*HHZ+HLOW
      JIND=+1
      DO 1001 KO=0,LIMV
      PH=FLOAT(KO)*HVT+VLOW
      CALLSUM(TH,PH,RZERO,XH,YH,ZH,WLAMDA,LMN,DELA,R,RMAX,XMAX,MRA,SHAD)
      IF(IREV .NE. 0) CALL REVB(REV,TH,PH)
      X=XMAX*COS(PH*DEG)/ARM
      IF(IREV .NE. 0) Y=REV*X
      IF(KO .NE. 0) GO TO 1002
 1003 DI=DI+X
      IF(IREV .NE. 0) RL=RL+Y
      GO TO 1001
 1002 IF(KO .EQ. LIMV) GO TO 1003
```

```
      DI=DI+X+X
      IF(IREV .NE. 0) RL=RL+Y+Y
      IF(JIND .GT. 0) DI=DI+X+X
      IF(JIND.GT.0 .AND. IREV.NE.0) RL=RL+Y+Y
      JIND=-JIND
1001  CONTINUE
      IF(JO .NE. 0) GO TO 1005
1004  DINDEX=DINDEX+FIVT*DI/3.
      IF(IREV .NE. 0) RLEVEL=RLEVEL+FIVT*RL/3.
      GO TO 1000
1005  IF(JO .EQ. LIMH) GO TO 1004
      DINDEX=DINDEX+2.*FIVT*DI/3.
      IF(IREV .NE. 0) RLEVEL=RLEVEL+2.*FIVT*RL/3.
      IF(IIND .GT. 0) DINDEX=DINDEX+2.*FIVT*DI/3.
      IF(IIND.GT.0.AND.IREV.NE.0) RLEVEL=RLEVEL+2.*FIVT*RL/3.
      IIND=-IIND
1000  CONTINUE
      TDI=TDI+FIHO2*DINDEX/3.0
      TRL=TRL+FIHO2*RLEVEL/3.0
      MANYH=MANYH-1
      GO TO 1210
1200  CONTINUE
      DINDEX=-10.*ALOG10(TDI/(4.*PI))
      IF(IREV.NE.0) RLEVEL=-10.*ALOG10(TRL/(4.*PI))
      IF(INDEX .NE. 0) WRITE (4,20) DINDEX
      IF(IREV .NE. 0) WRITE (4,21) RLEVEL
      GO TO 100
9999  CALL EXITG(DATA)
      STOP
      END
```

XYZ

```
      SUBROUTINE XYZ(IXYZ,XH,YH,ZH,L,M,N,XSPACE,YSPACE,ZSPACE,LMN,XO,YO,
     *ZO)
C **** A SUBROUTINE TO OBTAIN ARRAY ELEMENTS LOCATIONS
      DIMENSION XH(1),YH(1),ZH(1)
  100 FORMAT()
      LMN=L*M*N
      IF(IXYZ .NE. 0) GO TO 8
      IF(XSPACE .LE. 0.0) GO TO 3
C **** EQUALLY SPACED IN X-DIRECTION
      IXO=0
      XO=FLOAT(L-1)*XSPACE/2.0
      DO 1 I=1,L
      DO 11 J=1,M
      DO 21 K=1,N
      IXO=IXO+1
      XH(IXO)=FLOAT(I-1)*XSPACE-XO
   21 CONTINUE
   11 CONTINUE
    1 CONTINUE
    3 IF(YSPACE .LE. 0.0) GO TO 5
C **** EQUALLY SPACED IN Y-DIRECTION
      IYO=0
      YO=FLOAT(M-1)*YSPACE/2.0
      DO 4 J=1,M
      DO 14 I=1,L
      DO 24 K=1,N
      IYO=IYO+1
      YH(IYO)=FLOAT(M-1)*YSPACE-YO
   24 CONTINUE
   14 CONTINUE
    4 CONTINUE
    5 IF(ZSPACE .LE. 0.0) GO TO 6
C **** EQUALLY SPACED IN Z-DIRECTION
      IZO=0
      ZO=FLOAT(N-1)*ZSPACE/2.0
      DO 7 K=1,N
      DO 17 I=1,L
      DO 27 J=1,M
      IZO=IZO+1
      ZH(IZO)=FLOAT(K-1)*ZSPACE-ZO
   27 CONTINUE
   17 CONTINUE
    7 CONTINUE
    6 RETURN
C **** ELEMENTS UNEQUALLY SPACED
    8 CONTINUE
      XO=0.0
      YO=0.0
      ZO=0.0
      READ 100, LMN
      DO 9 I=1,LMN
      READ 100, XH(I),YH(I),ZH(I)
```

```
      XO=XO+XH(I)
      YO=YO+YH(I)
      ZO=ZO+ZH(I)
    9 CONTINUE
      XO=XO/FLOAT(LMN)
      YO=YO/FLOAT(LMN)
      ZO=ZO/FLOAT(LMN)
      DO 10 I=1,LMN
      XH(I)=XH(I)-XO
      YH(I)=YH(I)-YO
      ZH(I)=ZH(I)-ZO
   10 CONTINUE
      RETURN
      END
```

SUM

```
      SUBROUTINE SUM(THETA,PHI,RZERO,XH,YH,ZH,WLAMDA,LMN,DELA,R,RMAX,XMA
     *X,ISTEER,SHAD)
C **** PERFORM TRIPLE SUMMATION
      DIMENSION XH(1),YH(1),ZH(1),DELA(1),R(1),SHAD(1)
      DATA RAD,PI/.1745329E-01,6.283185307/
      TH=THETA*RAD
      PH=PHI*RAD
      ALPHA=COS(PH)*SIN(TH)
      BETA=COS(PH)*COS(TH)
      GAMMA=-SIN(PH)
      REAL=0.0
      XIM=0.0
      IF(ISTEER .EQ. 0) GO TO 104
      IF(RZERO .GT. 0.0) GO TO 40
C **** PLANE WAVE FRONT
      DO 103 I=1,LMN
      ARG=PI*(XH(I)*ALPHA+YH(I)*BETA+ZH(I)*GAMMA)/WLAMDA
      ARG=ARG+DELA(I)
      REAL=REAL+SHAD(I)*R(I)*COS(ARG)/RMAX
      XIM=XIM+SHAD(I)*R(I)*SIN(ARG)/RMAX
  103 CONTINUE
      GO TO 41
C **** COMPUTE STEERING DIRECTIONS
  104 CONTINUE
      DO 105 I=1,LMN
      ARG=PI*(XH(I)*ALPHA+YH(I)*BETA+ZH(I)*GAMMA)/WLAMDA
      DELA(I)=DELA(I)-ARG
  105 CONTINUE
      XMAX=FLOAT(LMN)**2
      ISTEER=1
      RETURN
C **** SPHERICAL WAVE FRONT
   40 CONTINUE
      ALPHA=ALPHA*RZERO
      BETA=BETA*RZERO
      GAMMA=GAMMA*RZERO
      DO 203 I=1,LMN
      ARG=(XH(I)-ALPHA)**2+(YH(I)-BETA)**2+(ZH(I)-GAMMA)**2
      ARG=PI*(RZERO-SQRT(ARG))/WLAMDA
      ARG=ARG+DELA(I)
      REAL=REAL+SHAD(I)*R(I)*COS(ARG)/RMAX
      XIM=XIM+SHAD(I)*R(I)*SIN(ARG)/RMAX
  203 CONTINUE
   41 XMAX=REAL*REAL+XIM*XIM
      RETURN
      END
```

BPLOTR

```
C  BEAM PLOT - RECTANGULAR
C
      SUBROUTINE BPLOTR(IBEAM,ANGLE,DL,NPTS,AFIX,AMIN,AMAX,ALONG,DLLONG,
     *HDR,DATA)
      DIMENSION ANGLE(1),DL(1),HDR(1),DATA(1)
      DATA RPI/310.2/
C
C  IBEAM NOT 0:  VERTICAL PATTERN
C         =  0:  HORIZONTAL PATTERN
C  AFIX:  FIXED ANGLE (DEG)
C  AMIN:  MINIMUM ANGLE (DEG)
C  AMAX:  MAXIMUM ANGLE (DEG)
C  ALONG:  LENGTH OF ANGLE AXIS (IN)
C  DLLONG:  LENGTH OF LOSS AXIS (IN)
C  HDR:  TITLE
C
      XMIN = (4095. - ALONG*RPI)/2.
      XMAX = XMIN + ALONG*RPI
      YMAX = 3071. - .5*RPI
      YMIN = YMAX - DLLONG*RPI
      CALL OBJCTG(DATA,XMIN,YMIN,XMAX,YMAX)
      DLMAX = 10.*DLLONG
      CALL SUBJEG(DATA,AMIN,DLMAX,AMAX,0.)
      ADEL = (AMAX-AMIN)/ALONG
      CALL GRIDG(DATA,ADEL,-10.,0,0)
      CALL LABELG(DATA,0,ADEL,0,6,1)
      CALL LABELG(DATA,1,-10.,0,3)
      CALL TITLEG(DATA,11,'ANGLE (DEG)',16,'DB DOWN FROM MRA',72,HDR)
      DO 20 I=1,NPTS
      IF(ANGLE(I).LT.AMIN)  ANGLE(I) = AMIN
      IF(ANGLE(1).GT.AMAX)  ANGLE(I) = AMAX
      IF(DL(I).LT.0.)  DL(I) = 0.
      IF(DL(I).GT.DLMAX)  DL(I) = DLMAX
   20 CONTINUE
      CALL SETSMG(DATA,30,2.)
      CALL LINESG(DATA,NPTS,ANGLE,DL)
      CALL SETSMG(DATA,30,1.)
      CALL SETSMG(DATA,14,1.)
      Y = YMIN - 1.25*RPI
      IF(IBEAM.EQ.0)  GO TO 50
      X = (4095. - 21.*31.)/2.
      CALL LEGNDG(DATA,X,Y,21,'VERTICAL BEAM PATTERN')
      Y = Y - .3*RPI
      X = (4095. - 18.*31.)/2.
      CALL LEGNDG(DATA,X,Y,18,'THETA =        DEG')
      X = X + 31.*8.
      GO TO 100
   50 X = (4095. - 23.*31.)/2.
      CALL LEGNDG(DATA,X,Y,23,'HORIZONTAL BEAM PATTERN')
      Y = Y - .3*RPI
      X = (4095. - 16.*31.)/2.
      CALL LEGNDG(DATA,X,Y,16,'PHI =        DEG')
      X = X + 6.*31.
  100 CALL NUMBRG(DATA,X,Y,6,1,AFIX)
      CALL SETSMG(DATA,14,0.)
      CALL PAGEG(DATA,0,1,1)
      RETURN
      END
```

BPLOTP

```
C   BEAM PLOT - POLAR
C
      SUBROUTINE BPLOTP(IBEAM,ANGLE,DL,NPTS,AFIX,DLMAX,DLLONG,HDR,DATA)
      DIMENSION ANGLE(1),DL(1),HDR(1),DATA(1)
      DATA RPI/310.7/
      DATA DEG/57.295779/
C
C   IBEAM NOT 0:  VERTICAL PATTERN
C         =   0:  HORIZONTAL PATTERN
C   AFIX:  FIXED ANGLE (DEG)
C   DLMAX:  RANGE OF DL (DB)
C   DLLONG:  RADIUS (IN)
C   HDR:  TITLE
C
      XMIN = 2047.5 - DLLONG*RPI
      XMAX = XMIN + 2.*DLLONG*RPI
      YMAX = 3071. - .5*RPI
      YMIN = YMAX - 2.*DLLONG*RPI
      CALL OBJCTG(DATA,XMIN,YMIN,XMAX,YMAX)
      CALL SUBJEG(DATA,-DLLONG,-DLLONG,DLLONG,DLLONG)
      CALL RADIAL(DLLONG+.12,DATA)
      CALL SUBJEG(DATA,-DLMAX,-DLMAX,DLMAX,DLMAX)
      CALL SETSMG(DATA,30,2.)
      CALL SETSMG(DATA,83,1.)
      CALL CIRARG(DATA,0.,0.,DLMAX,0.,360.)
      CALL SETSMG(DATA,30,1.)
      DO 10 I=0,170,10
      THETA = FLOAT(I)/DEG
      X = DLMAX*COS(THETA)
      Y = DLMAX*SIN(THETA)
   10 CALL SEGMTG(DATA,1,X,Y,-X,-Y)
      DO 20 I=1,10
      X = 10.*I
      IF(X.GE.DLMAX)  GO TO 30
   20 CALL CIRARG(DATA,0.,0.,DLMAX-X,0.,360.)
   30 CALL SETSMG(DATA,30,2.)
      THETA = (90. - ANGLE(1))/DEG
      R = DLMAX - DL(1)
      X = R*COS(THETA)
      Y = R*SIN(THETA)
      CALL LINESG(DATA,0,X,Y)
      DO 40 I=2,NPTS
      THETA = (90.-ANGLE(I))/DEG
      R = DLMAX - DL(I)
      X = R*COS(THETA)
      Y = R*SIN(THETA)
   40 CALL LINESG(DATA,1,X,Y)
      CALL SETSMG(DATA,30,1.)
      CALL SETSMG(DATA,14,1.)
      Y = YMIN - 160.
      X = (4095. - 72.*31.)/2.
      CALL LEGNDG(DATA,X,Y,72,HDR)
```

```
      Y = YMIN - 1.25*RPI
      IF(IBEAM.EU.0)  GO TO 50
      X = (4095. - 21.*31.)/2.
      CALL LEGNDG(DATA,X,Y,21,'VERTICAL BEAM PATTERN')
      Y = Y - .3*RPI
      X = (4095. - 18.*31.)/2.
      CALL LEGNDG(DATA,X,Y,18,'THETA =        DEG')
      X = X + 31.*6.
      GO TO 100
   50 X = (4095. - 23.*31.)/2.
      CALL LEGNDG(DATA,X,Y,23,'HORIZONTAL BEAM PATTERN')
      Y = Y - .3*RPI
      X = (4095. - 16.*31.)/2.
      CALL LEGNDG(DATA,X,Y,16,'PHI =        DEG')
      X = X + 6.*31.
  100 CALL NUMBRG(DATA,X,Y,6,1,AFIX)
      X=RPI*DLLONG/DLMAX*10.
      Y=YMAX+25.
      DO 120 I=1,10
      IX=10*I
      IF(FLOAT(IX) .GE. DLMAX) GO TO 130
      Y=Y-X
  120 CALL NUMBRG(DATA,2032.5,Y,3,IX)
  130 CONTINUE
      CALL SETSMG(DATA,14,0.)
      CALL PAGEG(DATA,0,1,1)
      RETURN
      END
```

RADIAL

```
      SUBROUTINE RADIAL(RAD,DATA)
      DIMENSION DATA(1)
      EXTERNAL FONT2
      DATA DEG/57.29578/
      CALL VECIG(DATA,FONT2,0)
      CALL SETSMG(DATA,51,1.)
      CALL NUMBRG(DATA,0.,RAD,1,0)
      DANG2 = 1./2.*31./(RAD*270.18)
      DANG3 = 2.*DANG2
      DO 1 K=10,90,10
      FPN = FLOAT(K)
      DANG = FPN/DEG - DANG2
      XO = RAD*SIN(DANG)
      YO = RAD*COS(DANG)
      THETA = -FPN
      CALL SETSMG(DATA,46,THETA)
      CALL NUMBRG(DATA,XO,YO,2,K)
    1 CONTINUE
      DO 2 K=100,260,10
      FPN = FLOAT(K)
      DANG = FPN/DEG + DANG3
      XO = RAD*SIN(DANG)
      YO = RAD*COS(DANG)
      THETA = 180. - FPN
      CALL SETSMG(DATA,46,THETA)
      CALL NUMBRG(DATA,XO,YO,3,K)
    2 CONTINUE
      DO 3 K=270,350,10
      FPN = FLOAT(K)
      DANG = FPN/DEG - DANG3
      XO = RAD*SIN(DANG)
      YO = RAD*COS(DANG)
      THETA = 360. - FPN
      CALL SETSMG(DATA,46,THETA)
      CALL NUMBRG(DATA,XO,YO,3,K)
    3 CONTINUE
      CALL SETSMG(DATA,51,0.)
      CALL SETSMG(DATA,46,0.)
      RETURN
      END
```

SHADE

```
      SUBROUTINE SHADE(LMN,SHAD,XH,YH,ZH,XO,YO,ZO)
      DIMENSION SHAD(1),XH(1),YH(1),ZH(1)
      PED=0.5
C **** COSINE SQUARE SHADING TECHNIQUE
      CAY1=ACOS(SQRT(0.1))/XO
      CAY3=ACOS(SQRT(0.1))/ZO
      DO 33 I=1,LMN
      XA=(1.-PED)*COS(CAY1*XH(I))**2+PED
      XB=(1.-PED)*COS(CAY3*ZH(I))**2+PED
      SHAD(I)=SHAD(I)*XA*XB
   33 CONTINUE
      RETURN
      END
```

DELAY

```
      SUBROUTINE DELAY(LMN,DELA)
C **** SELECTABLE PHASE DELAY IN RADIANS
      DIMENSION DELA(1)
  100 FORMAT()
      READ 100, (DELA(I),I=1,LMN)
      RETURN
      END
```

RSPNSE

```
      SUBROUTINE RSPNSE(LMN,R)
C **** RESERVED TO OBTAIN DIFFERENT RESPONSE OF DIFFERENT ELEMENTS
      DIMENSION R(1)
      RETURN
      END
```

REVB

```
SUBROUTINE REVB(X,TH,PH)
X=0.5
RETURN
END
```